Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at http://about.jstor.org/participate-jstor/individuals/early-journal-content.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

SOME MODERN ADVANCES IN LOGIC.

MATHEMATICS is traditionally supposed to be occupied with questions about number and quantity. During the last thirty years or so certain mathematicians —a German named Frege, an Italian named Peano, and later, in England, Mr. Bertrand Russell and Dr. A. N. Whitehead—have been studying this sort of question: Take any mathematical proposition; prove it carefully, that is to say write down completely all the logical steps by which that proposition follows from more simple ones; then enumerate completely the fundamental notions in terms of which the notions occurring in that proposition are defined, and the principles of inference used. Euclid attempted—in a way that to modern eyes is very unsatisfactory, whether we consider his tacit assumptions or his prolixity —to reduce the foundations of geometry to a set of definitions, postulates, and axioms. Euclid's definitions are often (as in the case of those of a point and a straight line) only would-be explanations of certain ideas which everybody is supposed to have, and which are really assumed as primitive notions which are a necessary preliminary to what follows. Further, Euclid does not reckon among his axioms and primitive ideas the principles, such as the syllogism, and the fundamental ideas of logic itself. He tacitly assumes these as preliminary to geometry.

Modern people have gone far beyond this. Peano's work, though in some ways not nearly so fundamental and subtle as Frege's, has become far better known than the German's. This is owing to the noble self-sacrifice of Peano himself. For years past he has spared neither time nor money in the editing and publishing of a journal and a periodical collection of mathematical propositions expressed in the symbolism partly invented by himself. We all know the appearance of mathematical symbols; and some of us know that the introduction of an analogous set of symbols has had incalculable benefit on other sciences, such as logic and chemistry. Peano's symbolism consists of certain very convenient signs for denoting logical notions, so that logical propositions can be translated into a form like that of mathematical equations; logical operations become easily and almost mechanically carried out, and it becomes possible to condense the expression of a long chain of reasoning into a short and readily grasped form.

The idea of such a language is not new. It goes back to Leibniz and Descartes, or perhaps earlier, and began to be vigorously developed about the middle of the nineteenth century by the English mathematicians Boole and De Morgan.

One result of Peano's work was the discovery that all the ideas which occur in arithmetic and geometry and the other sciences usually called mathematical can be defined in terms of the ideas of general logic, such as class, implication, membership of a class, aggregation and disjunction of classes, together with five or six other ideas, such as integer, number, and point. Also Peano's work contained contributions of the utmost importance to logic, such as the perception that inference in mathematics was not the inference of one proposition from another, but the inference of a whole class of propositions from another class.

Mr. Russell, partly helped by a study of Frege's work, and partly having discovered for himself many of Frege's distinctions, took up Peano's work where Peano had left it, and defined in logical terms alone all of Peano's fundamental mathematical ideas and proved all his fundamental mathematical propositions. Thus nowadays mathematics and logic are seen to form part of a continuous whole.

Further, it now appears that the essential character of mathematical propositions is not, as Euclid would have it, —"A is true, therefore B is true," but "if A is true, then B is true." In geometry, for example, we do not, as formerly everybody used to think, study the properties of the space we live in. We only say things of the form—"if space has such-and-such properties, then it has such-and-such other properties."

Mr. Russell's work, begun in 1900, now seems to be entering the stage of completion. Towards the end of last year the Cambridge University Press published the first volume of a treatise called *Principia Mathematica* by Messrs. Russell and Whitehead. Here are nearly 700 pages, written to a great extent in the modified Peano-symbolism and exposing in detail the modern views on logic and mathematics. Nowadays a mathematician will tell you that, of the two things with which tradition supposes mathematics to deal, number is definable in logical terms, so that mathematics is only a further-developed logic, and quantity is not considered at all. Serial order is, and people tend to confuse that with quantity.

PHILIP E. B. JOURDAIN.

BROADWINDSOR, BEAMINSTER, DORSET, ENGLAND.